Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Mechanics M4

Advanced/Advanced Subsidiary

Wednesday 15 June 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6680/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1.

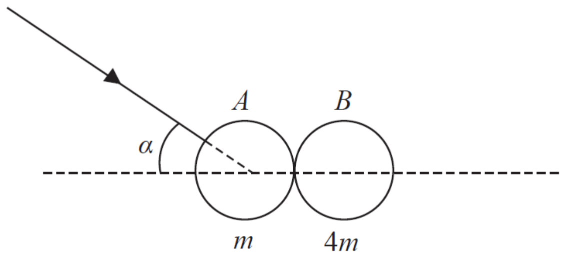

Figure 1

A smooth uniform sphere A of mass m is moving on a smooth horizontal plane when it collides with a second smooth uniform sphere B, which is at rest on the plane. The sphere B has mass $4m$ and the same radius as A. Immediately before the collision the direction of motion of A makes an angle α with the line of centres of the spheres, as shown in Figure 1. The direction of motion of A is turned through an angle of 90° by the collision and the coefficient of restitution between the spheres is $\frac{1}{2}$.

Find the value of tan α.

(Total 8 marks)

2.

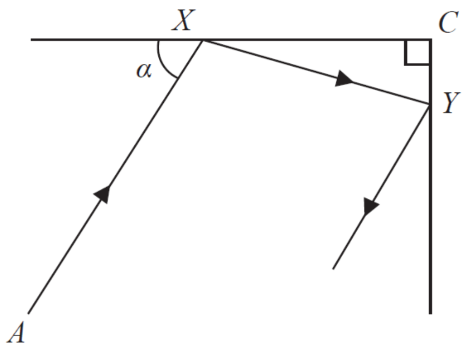

Figure 2

A small spherical ball P is at rest at the point A on a smooth horizontal floor. The ball is struck and travels along the floor until it hits a fixed smooth vertical wall at the point X. The angle between AX and this wall is α, where α is acute. A second fixed smooth vertical wall is perpendicular to the first wall and meets it in a vertical line through the point C on the floor. The ball rebounds from the first wall and hits the second wall at the point Y. After P rebounds from the second wall, P is travelling in a direction parallel to XA, as shown in Figure 2. The coefficient of restitution between the ball and the first wall is e. The coefficient of restitution between the ball and the second wall is ke.

Find the value of k.

(Total 9 marks)

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3. Two straight horizontal roads cross at right angles at the point X. A girl is running, with constant speed 5 m s^{-1}, due north towards X on one road. A car is travelling, with constant speed 20 m s^{-1}, due west towards X on the other road.

(*a*) Find the magnitude and direction of the velocity of the car relative to the girl, giving the direction as a bearing.

(6)

At noon the girl is 150 m south of X and the car is 800 m east of X.

(*b*) Find the shortest distance between the car and the girl during the subsequent motion.

(7)

(Total 13 marks)

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4. A particle P of mass 9 kg moves along the horizontal positive x-axis under the action of a force directed towards the origin. At time t seconds, the displacement of P from O is x metres, P is moving with speed v m s^{-1} and the force has magnitude $16x$ newtons. The particle P is also subject to a resistive force of magnitude $24v$ newtons.

(*a*) Show that the equation of motion of P is

$$9\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+24\frac{\mathrm{d}x}{\mathrm{d}t}+16x=0\,.$$

(4)

It is given that the general solution of this differential equation is

$$x=\mathrm{e}^{-\frac{4}{3}t}(At+B)\,,$$

where A and B are arbitrary constants.

When $t = \dfrac{3}{4}$, P is travelling towards O with its maximum speed of $8\mathrm{e}^{-1}$ m s^{-1} and $x = d$.

(*b*) Find the value of d.

(3)

(*c*) Find the value of x when $t = 0$

(5)

(Total 12 marks)

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5. A toy car of mass 0.5 kg is attached to one end A of a light elastic string AB, of natural length 1.5 m and modulus of elasticity 27 N. Initially the car is at rest on a smooth horizontal floor and the string lies in a straight line with AB = 1.5 m. The end B is moved in a straight horizontal line directly away from the car, with constant speed u m s^{-1}. At time t seconds after B starts to move, the extension of the string is x metres and the car has moved a distance y metres. The effect of air resistance on the car can be ignored.

By modelling the car as a particle, show that, while the string remains taut,

(*a*) (i) $x + y = ut$,

(2)

(ii) $\dfrac{d^2x}{dt^2} + 36x = 0$.

(4)

(*b*) Hence show that the string becomes slack when $t = \dfrac{\pi}{6}$.

(3)

(*c*) Find, in terms of u, the speed of the car when $t = \dfrac{\pi}{12}$.

(3)

(*d*) Find, in terms of u, the distance the car has travelled when it first reaches end B of the string.

(5)

(Total 17 marks)

6.

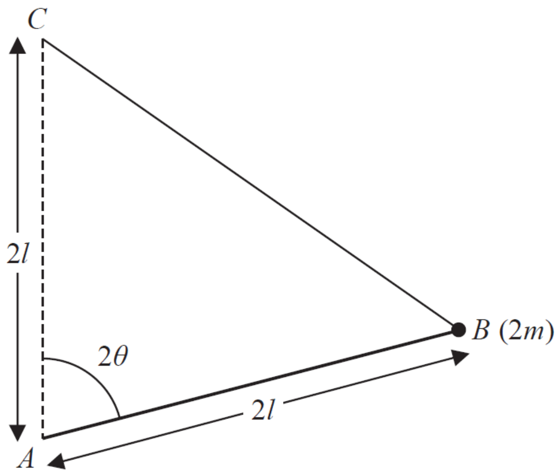


Figure 3

Figure 3 shows a uniform rod AB, of length $2l$ and mass $4m$. A particle of mass $2m$ is attached to the rod at B. The rod can turn freely in a vertical plane about a fixed smooth horizontal axis through A. One end of a light elastic spring, of natural length $2l$ and modulus of elasticity kmg, where $k > 4$, is attached to the rod at B. The other end of the spring is attached to a fixed point C which is vertically above A, where $AC = 2l$. The angle BAC is 2θ, where $\frac{\pi}{6} < \theta \leq \frac{\pi}{2}$.

(*a*) Show that the potential energy of the system is

$$4mgl\{(k-4)\sin^2\theta - \sin\theta\} + \text{constant}.$$

(6)

Given that there is a position of equilibrium with $\theta \neq \frac{\pi}{2}$,

(*b*) show that $k > 8$.

(6)

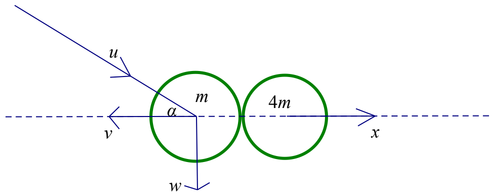
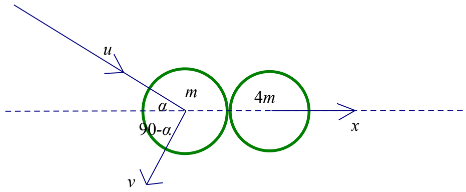
Given that $k = 10$,

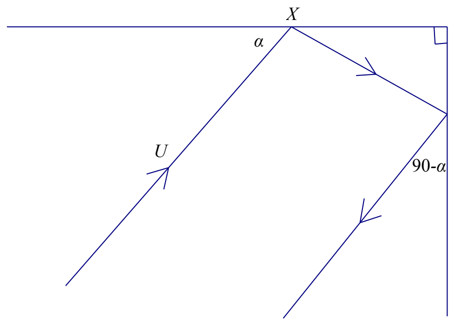
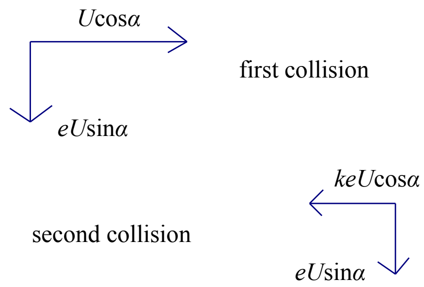
(*c*) determine the stability of this position of equilibrium.

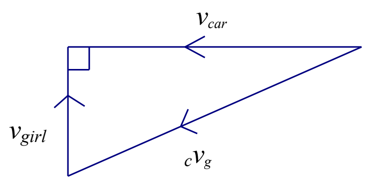
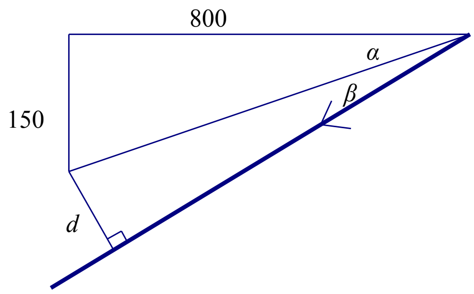
(4)

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

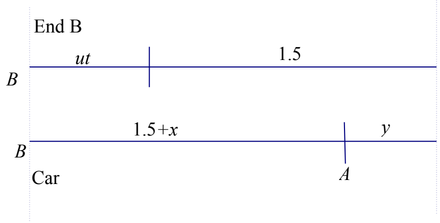
Q	Scheme	Marks	Notes
1.			
	Along line of centres:		
	Con of mom: $mu\cos\alpha = 4mx - mv$	M1	$mu\cos\alpha = 4mx - mv\cos\beta$ or $mu\cos\alpha = 4mx - mv\sin\alpha$ Need to see all 3 terms, but condone sign errors & trig. confusion
	$(u\cos\alpha = 4x - v)$	A1	$(u\cos\alpha = 4x - v\cos\beta)$ $(u\cos\alpha = 4x - v\sin\alpha)$
	NLR: $\frac{1}{2}u\cos\alpha = x + v$	M1	$\frac{1}{2}u\cos\alpha = x + v\cos\beta$ $\frac{1}{2}u\cos\alpha = x + v\sin\alpha$ Must be used the right way round, but condone sign errors & consistent trig. confusion
	$(2u\cos\alpha = 4x + 4v)$	A1	$(2u\cos\alpha = 4x + 4v\cos\beta)$ $(2u\cos\alpha = 4x + 4v\sin\alpha)$
	$(5v = u\cos\alpha)$		$(5v\tan\alpha = u)$ $(u\cos\alpha = 5v\cos\beta)$
	Perp to line of centres: no change to velocity so vel $= w = u\sin\alpha$	B 1 (A1)	$v\cos\alpha = u\sin\alpha$ $(v = u\tan\alpha)$
	Deflected through $90°$ $\left(\tan\alpha = \frac{v}{w}\right)$	B1	$90°$ used correctly. E.g. use of $90 - \alpha$ in an equation $(\tan\alpha \times \tan\beta = 1)$
	$\tan\alpha = \dfrac{\frac{1}{5}u\cos\alpha}{u\sin\alpha}$	M1	$5u\tan^2\alpha = u$ Form equation in α
	$\tan^2\alpha = \frac{1}{5}$ $\tan\alpha = \sqrt{\frac{1}{5}}$ or 0.4472....	A1	(0.45 or better)
		[8]	

Q	Scheme	Marks	Notes
2			
	First impact:		
	Component parallel to wall: $=U\cos\alpha$	B1	
	Perp to wall: NLR: $eU\sin\alpha$	M1	Correct use of impact law Condone trig. confusion
		A1	
	Second impact:		
	parallel to wall vel after $=eU\sin\alpha$	B1	In terms of U and α
	Perp to wall $ke\times U\cos\alpha$	B1	In terms of U and α
	Direction at $(90-\alpha)$ to the wall	B1	Seen or implied
	$\Rightarrow \tan(90-\alpha)=\dfrac{keU\cos\alpha}{Ue\sin\alpha}$ or $\tan\alpha=\dfrac{eU\sin\alpha}{keU\cos\alpha}$	M1	
	$\cot\alpha=k\cot\alpha$ or $\tan\alpha=\dfrac{1}{k}\tan\alpha$	A1	Equation in k and α
	$k=1$	A1	From correct work only
		[9]	
	NB:A candidate who makes a false assumption about an angle α in triangle CXY can score a maximum B1B1B1 B0B0 B1 M1 A0 (6/8)		

Q	Scheme	Marks	Notes
3			
(a)	${}_c\mathbf{v}_g = \mathbf{v}_c - \mathbf{v}_g$ 	B1	Correct vector triangle seen or implied e.g. sight of ${}_c\mathbf{v}_g = \begin{pmatrix} -20 \\ -5 \end{pmatrix}$ or correct final bearing
	$\tan\theta = \frac{20}{5}$ $\theta = 75.96°$ or $\tan\theta = \frac{5}{20}$, $\theta = 14.04°$	M1	Use trig. to find a relevant angle
		A1	Angle correct
	Direction is $256°$	A1	
	Mag $= \sqrt{20^2 + 5^5}$	M1	
	$= \sqrt{425}$ $(= 20.61...)$ (m s^{-1})	A1	$5\sqrt{17}$ Accept 21
		(6)	
(b)			
	Dist apart at noon $= \sqrt{150^2 + 800^2}\left(= \sqrt{662500} = 813.94...\right)$	M1	
	$\tan\alpha = \frac{150}{800}$	M1	Use trig to find α
	$\alpha = \tan^{-1}\left(\frac{150}{800}\right)$, $(\alpha = 10.619)$	A1	Correct equation in α
	$\beta = 14.04 - 10.619 = 3.420...$	M1 (A1)	Correct strategy for β Their θ – their α
	$\sin\beta = \frac{d}{\sqrt{662500}}$	M1	Use trig. to find d
		A1	Correct unsimplified expression
	$d = \sqrt{662500}\sin 3.42$ $= 48.5...$ m	A1	or exact answer $\frac{200}{\sqrt{17}}$ Accept 49 or better
		(7)	
		[13]	
			See over for vector alternative

Q	Scheme	Marks	Notes
3balt	Relative position $\begin{pmatrix} 800-20t \\ 150-5t \end{pmatrix}$	M1	By subtraction
	Distance $d^2 = (800-20t)^2 + (150-5t)^2$	M1	Correct use of Pythagoras' theorem
	$\left(= 425t^2 - 33500t + 662500\right)$	A1	Correct unsimplified expression for d or d^2
	$-40(800-20t) - 10(150-5t)$	M1	Differentiate
	$(850t - 33500 = 0)$	M1 (A1)	Equate to zero and solve for t.
	$t = \dfrac{670}{17}$, 39.4(s)	A1	
	$\Rightarrow d = 48.5$ (m)	A1	
3balt	Relative position $\begin{pmatrix} 800-20t \\ 150-5t \end{pmatrix}$	M1	By subtraction
	Distance $d^2 = (800-20t)^2 + (150-5t)^2$	M1	Correct use of Pythagoras' theorem
	$\left(= 425t^2 - 33500t + 662500\right)$	A1	Correct unsimplified expression for d or d^2
	$\begin{pmatrix} 800-20t \\ 150-5t \end{pmatrix} \bullet \begin{pmatrix} -20 \\ -5 \end{pmatrix} = 0$	M1	Use scalar product with ${}_c\mathbf{v}_g = \begin{pmatrix} -20 \\ -5 \end{pmatrix}$
	$-20(800-20t) - 5(150-5t) = 0$	M1	Equate scalar product to zero and solve for t
	$t = \dfrac{670}{17}$, 39.4(s)	A1	
	$\Rightarrow d = 48.5$ (m)	A1	
4			

Q	Scheme	Marks	Notes
(a)	NL2: $9\dfrac{d^2x}{dt^2}=-24v-16x$	M1	Requires all 3 terms but condone sign errors. Condone $\dot{x}$ for v. Must be dimensionally correct.
		A1	Correct unsimplified equation with v. Accept with $9a$ in which case accept $\pm$
	$9\dfrac{d^2x}{dt^2}+24\dfrac{dx}{dt}+16x=0$	M1	Substitute for v (seen anywhere) to form equation in x and t only
		A1	**Given answer** as printed - from correct solution.
	NB: If never see v used, max score 1/4	(4)	
(b)	$\ddot{x}=0 \Rightarrow 16x=-24\dot{x}$	M1	$\ddot{x}=0$ used Accept equivalent forms
	$16d=24\times 8e^{-1}$	A1	$\dot{x}$ substituted correctly
	$d=12e^{-1}$	A1	4.4 or better
		(3)	
(b) alt		M1	Differentiate twice and find A and B. Condone use of $t=\dfrac{3}{4},\ \dot{x}=8e^{-1}$
	$x=e^{-\frac{4}{3}t}(8t+6)$	A1	
	$d=12e^{-1}$	A1	
		(3)	
(c)	$\dot{x}=-\dfrac{4}{3}e^{-\frac{4}{3}t}(At+B)+Ae^{-\frac{4}{3}t}$	M1	Differentiate the given general solution using the product rule
		A1	Correct unsimplified
	$-8e^{-1}=-\dfrac{4}{3}e^{-1}\left(\dfrac{3}{4}A+B\right)+Ae^{-1}$	M1	Use $t=\dfrac{3}{4},\ \dot{x}=-8e^{-1}$
	$-8=-A-\dfrac{4}{3}B+A$		
	$B=6$	A1	
	The first 4 marks for (c) are available when seen		
	$\left(x=e^{-\frac{4}{3}t}(At+6)\right)$		
	$t=0\quad x=6$	B1ft	their B
		(5)	
		[12]	

Q	Scheme	Marks	Notes
5 (a)(i)	Dist moved by end $B = ut$ Dist of end B from initial position of car $= 1.5 + ut$ Length of rope $= 1.5 + x$ $\therefore 1.5 + ut = 1.5 + x + y$	M1	
	$\Rightarrow x + y = ut$	A1	**Given answer**
		(2)	
(ii)	$T = \dfrac{27x}{1.5} = 18x$	B1	
	Eqn of motion for car: $0.5\dfrac{\mathrm{d}^2y}{\mathrm{d}t^2} = 18x$	M1	Must start out with $\ddot{y}$
	$x + y = ut \qquad -\ddot{x} = \ddot{y}$	A1	Correct substitution for $\ddot{y}$ - must be explained
	$(\ddot{x} = -36x) \qquad \Rightarrow \dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} + 36x = 0$	A1	No errors seen. **Given answer** as printed
		(4)	
(b)	$x = a\sin 6t$	M1	
	$x = 0 \quad \sin 6t = 0$	M1	Find the value of t when $x = 0$ or substitute $t = \dfrac{\pi}{6}$
	$6t = \pi \qquad t = \dfrac{\pi}{6}$	A1	No errors seen **Given answer**
		(3)	
(c)	$\dot{x} = 6a\cos 6t$	M1	Differentiate their x
	$\Rightarrow \dot{y} = u - 6a\cos 6t \ , \qquad u = 6a$	M1 (A1)	use $\dot{y} = u - \dot{x}$ and $\begin{cases} t = 0,\ \dot{y} = 0 \\ \text{or } t = \dfrac{\pi}{12} \end{cases}$
	$t = \dfrac{\pi}{12} \quad \dot{y} = u - u\cos\dfrac{6\pi}{12} = u$	A1	
		(3)	
(d)	String slack when $t = \dfrac{\pi}{6} \qquad \dot{y} = u - u\cos\pi$	M1	Find speed of car when string goes slack.
	$= 2u$	A1	
	Time $= 1.5 \div u = \dfrac{3}{2u}$	B1	Time to close gap $= \dfrac{1.5}{2u - u}$
	Total distance travelled $= \left(\dfrac{\pi}{6} + \dfrac{3}{2u}\right)u + 1.5$	M1	Distance travelled by B + 1.5
	$= \dfrac{\pi u}{6} + 3$	A1	
		(5)	
		[17]	

Q	Scheme	Marks	Notes
6 (a)	GPE $= 4mgl\cos 2\theta + 2mg \times 2l\cos 2\theta$	M1	GPE of rod + particle (relative to a fixed point)
	$= 8mgl\cos 2\theta$	A1	Correct total
	Length of string $= 2\times 2l\sin\theta$ EPE $= \dfrac{kmg}{4l}\left(4l\sin\theta - 2l\right)^2$	M1	Use of EPE $= \dfrac{\lambda x^2}{2a}$
	$= kmgl\left(2\sin\theta - 1\right)^2$	A1	
	$kmgl\left(4\sin^2\theta - 4\sin\theta + 1\right)$ $+8mgl\left(1-2\sin^2\theta\right) + \text{const}$	M1	Total PE expressed in $\sin\theta$
	$= 4mgl\left\{\left(k-4\right)\sin^2\theta - k\sin\theta\right\} + \text{const}$	A1	**Given answer** as printed
		(6)	
(b)	$V' = \left(4mgl\right)\left\{\left(k-4\right)2\sin\theta\cos\theta - k\cos\theta\right\}$	M1	Differentiate V – condone errors but do not accept integration
	$\Rightarrow V' = 0 \quad \left(k-4\right)2\sin\theta\cos\theta - k\cos\theta = 0$	DM1	Set $V' = 0$ and solve for $\text{trig}\left(\theta\right) = \text{f}\left(k\right)$ Dependent on the first M1
	$4\cos\theta\left\{\left(2k-8\right)\sin\theta - k\right\} = 0$		
	$\sin\theta = \dfrac{k}{2k-8}$	A1	$\cos\theta = 0 \Rightarrow \theta = \dfrac{\pi}{2}$ need not be seen
	$\dfrac{\pi}{6} < \theta < \dfrac{\pi}{2} \Rightarrow \left(\dfrac{1}{2} <\right)\dfrac{k}{2k-8} < 1$	B1ft	ft on their $\sin\theta$
	$\Rightarrow k < 2k-8$	M1	Solve right hand inequality for k
	$\therefore k > 8$	A1	**Given answer**
		(6)	
(c)	$V'' = \left(4mgl\right)\left(12\cos 2\theta + 10\sin\theta\right)$	M1	Substitute $k = 10$ and find second derivative of V
	$V'' = 8\left(mgl\right)\left(6\cos 2\theta + 5\sin\theta\right)$	A1	Any equivalent form
	$\sin\theta = \dfrac{10}{12} = \dfrac{5}{6} \qquad \cos 2\theta = 1 - 2\times\dfrac{25}{36} = -\dfrac{7}{18}$	DM1	Dependent on the previous M1 Substitute their trig. values Need to be considering the whole of V''
	$V'' = 8mgl\left(-\dfrac{42}{18} + \dfrac{25}{6}\right) = mgl\dfrac{44}{3} > 0$		Accept 14.3 or better
	$\therefore$ (V min and) equilibrium is stable.	A1	With no errors seen
		(4)	
		[16]	